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SYSTEMS ANALYSIS DIRECTORATE

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This monthly publication contains Memoranda for Record (MFR's) and other technical information that summarize the activities of the Systems Analysis Directorate, US Army Armament Materiel Readiness Command, Rock Island, IL. (The most significant MFR's and other data will be published as notes or reports at a later date.) The subjects dealt with are: Cannon-Launched Guided Projectile, Copperhead, and Ammunition Quality Levels and Effectiveness.		

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\* Memoranda for Record and other technical information are grouped according to subject when applicable, and in chronological order.

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DEPARTMENT OF THE ARMY  
HEADQUARTERS, UNITED STATES ARMY ARMAMENT/COMMAND MATERIEL READINESS  
ROCK ISLAND, ILLINOIS 61201

REPLY TO  
ATTENTION OF:

DRSAR-SA

22 Apr 77

(SUBJECT: Communications Studies Relative to the Copperhead (CLGP) System)

Mr. Lee Waters  
Martin Marietta Aerospace (Orlando Division)  
M. P. - 334  
P. O. Box 5837  
Orlando, FL 32805

Dear Mr. Waters:

References:

- a. FONECON, 19 Apr 77, between Mr. L. Waters (MMA) and Mr. G. Schlenker (DRSAR-SAM), subject: Communications Studies Relative to the Copperhead (CLGP) System.
- b. ARMCOM Draft Technical Report, Jan 74, title: Field Artillery Communications Studies With Application to CLGP (Inclosure 1).
- c. Test Report No. FM255, Headquarters MASSTER (Ft. Hood, TX), Jan 75, title: Forward Observer Team Equipped With Ground Laser Locator Designator (FOTEGLLD).
- d. Technical Report prepared by University of Oklahoma Research Institute, CDOG Project 65-2, Feb 66, title: Communications Traffic Analysis of the Fire Support System of Command and Control Information System.
- e. Technical Report, US Army TRADOC Systems Analysis Activity, Jan 75, with Addendum dated Mar 75, title: Cannon Launched Guided Projectile Communications Study.

Pursuant to the Ref a conversation, DRSAR-SA transmits herewith the Ref b document as requested. It is noted that this request has the concurrence of DRCPM-CAWS.

The Ref b study was performed before the CLGP COEA was conducted. Study results were presented to the CLGP Study Advisory Group (SAG)

DRSAR-SA

SUBJECT: Communications Studies Relative to the Copperhead (CLGP) System

and to an interested DA staff group. The principal outcome of this study was an increased attention to the communications needs of CLGP.

A direct manifestation of this improved awareness is the special communications procedures used in the CLGP System Field Test, called FOTEGLLD. (Ref c). This test showed that by modifying the fire message format and commo procedures the present communications network was not overloaded. Several factors are responsible for this conclusion:

a. The message arrival rate in FOTEGLLD was significantly smaller than ( $<1/2$ ) that studied in Ref b, which is taken as a worst case under surge conditions.

b. Shortened message format produced a mean message turnaround or service time of about 14 seconds.

c. A ten second slack interval was permitted to alert the FO to base, as suggested by Ref b.

For your information note Ref d, an early commo study focused at the battalion level with implications for automation. Also note that TRASANA performed studies relative to Copperhead communications (Ref e), subsequent to the ARMCOM work.

Sincerely yours,

1 Incl  
as

OTTO F. HAASE, JR.  
Acting Director  
Systems Analysis Directorate

CF:

Project Manager, Cannon Artillery Weapons Systems, ATTN: DRCPM-CAWS/  
Mr. Ed Manley/Mr. Ernie Zimpo/Mr. Jim Williams, Dover, NJ 07801  
Director, US Army Materiel Systems Analysis Activity, ATTN: DRXSY-GS/  
Mr. Dave Barnhardt, Aberdeen Proving Ground, MD 21005

FIELD ARTILLERY COMMUNICATIONS STUDIES  
WITH APPLICATION TO CLGP

by

George Schlenker

U.S. Army Armaments Command

Rock Island, Illinois

January 1974

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SOLUTION OF A MESSAGE QUEUING PROBLEM  
APPLIED TO BATTALION-LEVEL FIELD ARTILLERY FIRE CONTROL

Introduction

Field artillery forward observers (FOs) are presumed to communicate with a fire direction center (FDC) at artillery battalion level using FM radio. However, there are, in general, more FOs than FM channels or discrete frequency bands. A typical situation is nine FOs and three channels. The number of FOs using the three FM channels is variable, depending upon the availability of telephone lines and their operability status. In the following analysis the number of users of the commo net is a discrete variable  $N$ .

In using the commo net each of the FOs is regarded as a customer competing for service by an FM channel. The service time, therefore, reflects the time required by the FO to transmit a fire message and receive confirmation by the FDC. For several reasons this service time is a random variable whose probability distribution is approximately gamma with shape parameter 2, i.e.,  $\gamma(2)$ . A fundamental property of this distribution is that it can be considered as the distribution of the sum of two identically-distributed exponential variables. Thus each channel of service is regarded as two stages of service connected in tandem each stage of which has a service time which is exponentially distributed. The service rate per stage of service is designated as  $\mu$ . Since the number of customers using the commo net is strictly finite and small, a customer using or wanting to use the net must be removed from the population which generates message requests. To treat this situation,



each FO is regarded as a request generator with arrival rate  $\lambda$ . Therefore the total arrival rate for requests is

$$(N - n) \lambda$$

where  $n$  is the number of FOs in the service system, i.e., either using a channel or "in a queue" waiting to use a channel.

The problem studied here is a quantitative description of the above queuing process.

#### Definition of States of the System

Define an array of numbers such as  $x$  with the following format.

Queue length	Channel	Stage of Service	
		$S_1$	$S_2$
$x$	$C_1$	$x$	$x$
	$C_2$	$x$	$x$
	$C_3$	$x$	$x$

Thus the array

0	1	0
	0	1
	0	0

represents: zero queue, one customer in the first stage of service of the first channel, one customer in the second stage of the second channel, and no customers using the third channel.

With this format the states are enumerated and labeled in Figure 1.

Figure 1  
States of the Service System

Label	Configuration	Customers Present in the System
0	0 0 0 0 0 0 0	0
1.1	0 1 0 0 0 0 0	1
1.2	0 0 1 0 0 0 0	1
2.1	0 1 0 1 0 0 0	2
2.2	0 0 1 1 0 0 0	2
2.3	0 0 1 0 1 0 0	2
3.1	0 1 0 1 0 1 0	3
3.2	0 0 1 1 0 1 0	3
3.3	0 0 1 0 1 1 0	3
3.4	0 0 1 0 1 0 1	3
4.1	1 1 0 1 0 1 0	4
4.2	0 0 1 1 0 1 0	4

Figure 1  
(Continued)

Label	Configuration	Customers Present in the System
4.3	1 0 1 0 1 1 0	4
4.4	1 0 1 0 1 0 1	4

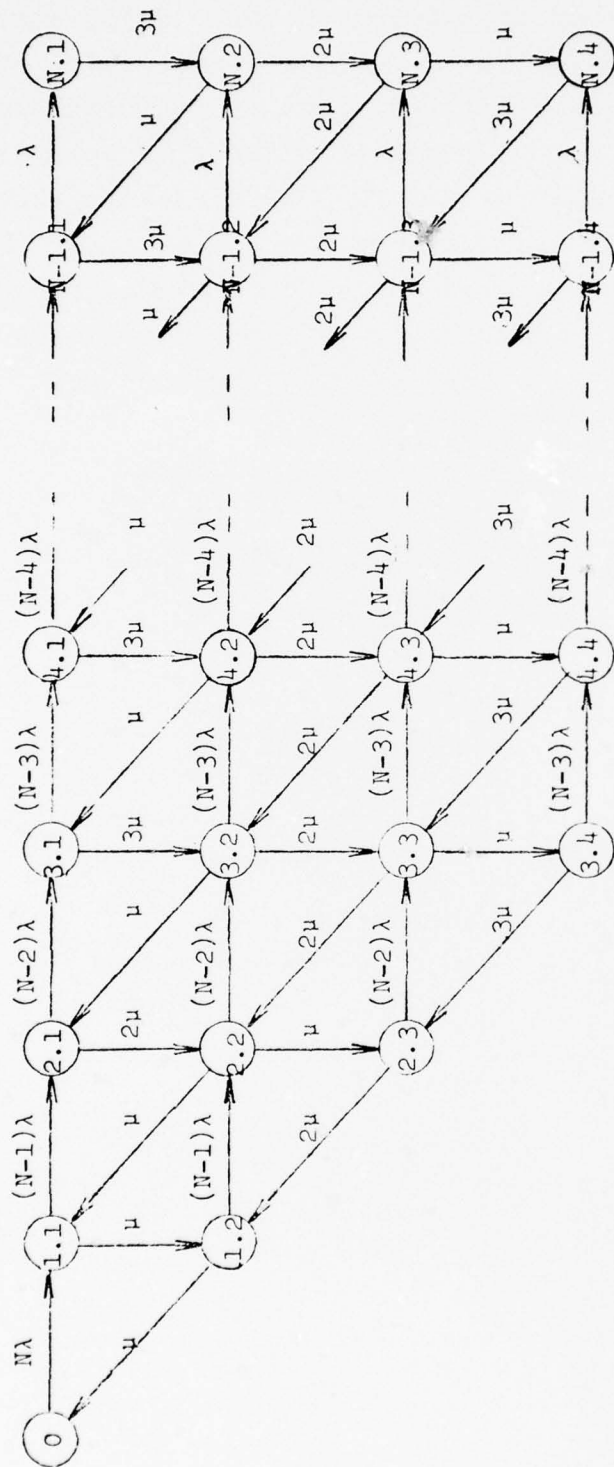
In general, for  $j$  customers present in the system,  
 $3 \leq j \leq N$ ,

j.1	j-3 1 0 1 0 1 0	j
j.2	j-3 0 1 1 0 1 0	j
j.3	j-3 0 1 0 1 1 0	j
j.4	j-3 0 1 0 1 0 1	j

Changes or transitions between these states occur stochastically. The conditional probabilities for the occurrence of these transitions in a differential time interval  $dt$  can be represented in a State Transition Diagram shown in Figure 2. The direction taken by the transition is indicated by the direction of the arrow (arc) between labeled nodes, each of which represents a state. The first-order rate parameters for the transitions are placed adjacent to the proper arcs.



Figure 2  
State Transition Diagram



### State Equations

The set of differential equations describing the dynamics of this service system can be written by inspecting the state transition diagram. In writing the differential equation for the probability of being in state  $n$  at time  $t$ ,  $P_n(t)$ , one examines arcs at the  $n$ th node. One simply assigns the rate parameter of a particular arc entering that node as a positive coefficient to the probability associated with the origin of that arc. For arcs leaving the  $n$ th node, a negative value is assigned the rate parameter(s) which is (are) used as coefficient(s) of  $P_n(t)$ . Thruout the network, rate parameters are treated as proportionality constants of flow rates and Kirchoff's law is satisfied. As a consequence of satisfying Kirchoff's law, the sum of all elements in each column of the coefficient matrix of the differential equations must equal zero. The last property can be used to check the validity of the differential equations.

In the following equations we omit the period in the state labels used in Figure 1. Additionally the functional dependence on time of the probabilities has been suppressed.

$$\dot{P}_0 = -N\lambda P_0 + \mu P_{12}$$

$$\dot{P}_{11} = N\lambda P_0 - [(N-1)\lambda + \mu] P_{11} + \mu P_{22}$$

$$\dot{P}_{12} = \mu P_{11} - [(N-1)\lambda + \mu] P_{12} + 2\mu P_{23}$$

$$\dot{P}_{21} = (N-1)\lambda P_{11} - [(N-2)\lambda + 2\mu] P_{21} + \mu P_{32}$$

$$\dot{P}_{22} = (N-1)\lambda P_{12} + 2\mu P_{21} - [(N-2)\lambda + 2\mu] P_{22} + 2\mu P_{33}$$

$$\dot{P}_{23} = \mu P_{22} - [(N-2)\lambda + 2\mu] P_{23} + 3\mu P_{34}$$

$$\dot{P}_{31} = (N-2)\lambda P_{21} - [(N-3)\lambda + 3\mu] P_{31} + \mu P_{42}$$

$$\begin{aligned} \dot{P}_{32} = 3\mu P_{31} + (N-2)\lambda P_{22} - [(N-3)\lambda + 3\mu] P_{32} \\ + 2\mu P_{43} \end{aligned}$$

$$\begin{aligned} \dot{P}_{33} = 2\mu P_{32} + (N-2)\lambda P_{23} - [(N-3)\lambda + 3\mu] P_{33} \\ + 3\mu P_{44} \end{aligned}$$

$$\dot{P}_{34} = \mu P_{33} - [(N-3)\lambda + 3\mu] P_{34}$$

For a number in the system  $n$  with

$$4 \leq n \leq N - 1$$

$$\begin{aligned} \dot{P}_{10n+1} = (N-n+1)\lambda P_{10(n-1)+1} \\ - [(N-n)\lambda + 3\mu] P_{10n+1} \\ + \mu P_{10(n+1)+2} \end{aligned}$$

$$\begin{aligned} \dot{P}_{10n+2} = 3\mu P_{10n+1} + (N-n+1)\lambda P_{10(n-1)+2} \\ - [(N-n)\lambda + 3\mu] P_{10n+2} \\ + 2\mu P_{10(n+1)+3} \end{aligned}$$

$$\begin{aligned}\dot{P}_{10n+3} &= 2\mu P_{10n+2} + (N-n+1)\lambda P_{10(n-1)+3} \\ &\quad - [(N-n)\lambda + 3\mu] P_{10n+3} \\ &\quad + 3\mu P_{10(n+1)+4}\end{aligned}$$

$$\begin{aligned}\dot{P}_{10n+4} &= \mu P_{10n+3} + (N-n+1)\lambda P_{10(n-1)+4} \\ &\quad - [(N-n)\lambda + 3\mu] P_{10n+4}\end{aligned}$$

The last four states of the system are described by

$$\dot{P}_{10N+1} = \lambda P_{10(N-1)+1} - 3\mu P_{10N+1}$$

$$\dot{P}_{10N+2} = 3\mu P_{10N+1} + \lambda P_{10(N-1)+2} - 3\mu P_{10N+2}$$

$$\dot{P}_{10N+3} = 2\mu P_{10N+2} + \lambda P_{10(N-1)+3} - 3\mu P_{10N+3}$$

$$\dot{P}_{10N+4} = \mu P_{10N+3} + \lambda P_{10(N-1)+4} - 3\mu P_{10N+4} \quad (1)$$

#### Numerical Formalism

In using a computer program to solve the above differential equations, it is necessary to place the state variables in a sequential array  $U(i,j)$ , where  $i$  is indexed from one to the number of states describing the system and where  $j = 1$  for the state probability and  $j = 2$  for the first derivative of the state probability.

The number of states for the system, i.e., the maximum value of the index  $i$  above, depends upon the number of customers  $N$  as follows.



$$\max(i) = 4 N - 2 \quad (2)$$

In the computer program developed by Stuart Olson (WECOM Report No. PAA-TRI-72), [1] an array is required for work space  $W(k)$ . The dimension of  $k$  in this array is given as

$$\max(k) = 3 \max(i)/2$$

$$\max(k) = 3 (2 N - 1) \quad (3)$$

An association between the mathematical notation

$$P_{ij}(t)$$

and the computer-oriented notation

$$U(k,1)$$

is provided below.

$$U(1,1) = P_0$$

$$U(2,1) = P_{11}$$

$$U(3,1) = P_{12}$$

$$U(4,1) = P_{21}$$

$$U(5,1) = P_{22}$$

$$U(6,1) = P_{23}$$

Generally, for  $k \geq 7$ ,

$$k = 4i + j - 6 \quad \text{so that}$$

$$U(4i + j - 6, 1) = P_{ij}, \quad \begin{matrix} 3 \leq i \leq N, \\ 1 \leq j \leq 4. \end{matrix} \quad (4)$$

#### Collections of States

As implied in Figure 1, certain states may be treated collectively to describe a measurable property of the system. For example, the probability of finding  $n$  customers present in the service system is obtained as follows

$$P\{n \text{ in the system}\} = p_n \quad \text{with}$$

$$p_0 = U(1,1)$$

$$p_1 = U(2,1) + U(3,1)$$

$$p_2 = U(4,1) + U(5,1) + U(6,1)$$

$$p_n = \sum_{j=1}^4 U(j+n+3,1), \quad 3 \leq n \leq N. \quad (5)$$

The number in the queue, i.e., waiting to capture a channel, is just three less than the number in the system when this is three or greater. The expected number in the queue

$$E[Q] = \sum_{n=4}^N (n-3) p_n \quad (6)$$

The probability that the queue length is zero is given as

$$P\{\text{zero queue}\} = \sum_{k=1}^{10} U(k,1) \quad . \quad (7)$$

### Parameter Values

In many tactical situations the message arrival rate within the commo system will be time-dependent. It is nevertheless instructive to examine system dynamics in response to a sudden initial arrival rate,  $\lambda$ , held constant thereafter. The system is assumed to be vacant initially. All results presented in the following graphs were obtained for  $\lambda = 0.01667 \text{ sec}^{-1}$ . The corresponding mean time between arrivals per FO (MTBA) of 60 seconds is regarded as the shortest interval of interest.

An interesting range of service rates for the CLGP system is from 0.04 to 0.10  $\text{sec}^{-1}$ . These correspond to mean times to serve (MTTS) of 50 and 20 sec., respectively [2].

### Results and Interpretation

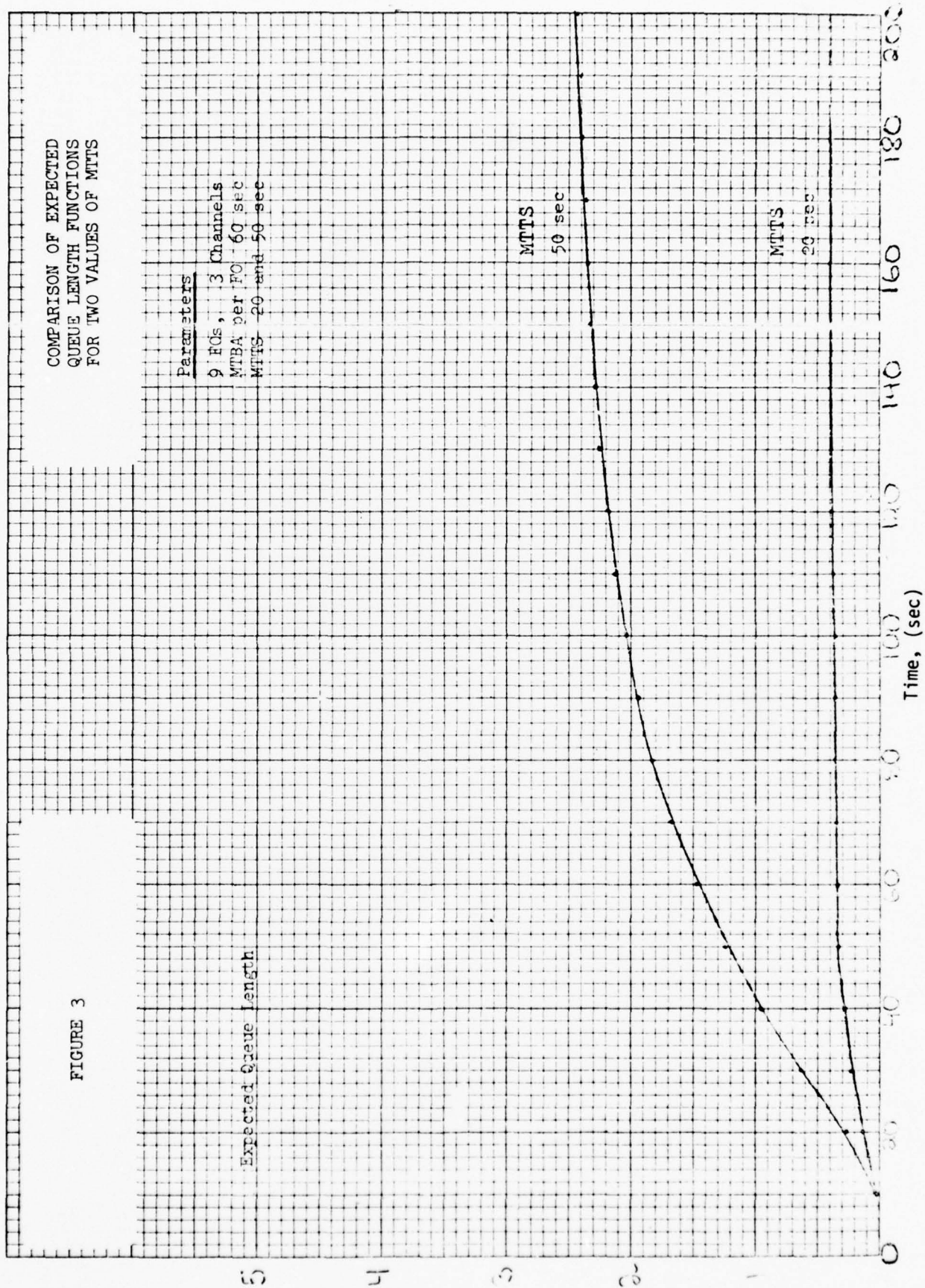
Using expected queue length,  $E[Q]$ , as a measure of the state of the service system, one can compare the approach toward stationary queuing by inspecting a plot of  $E[Q]$  versus time. This type of function is displayed in Figure 3 for the case of 9 FOs and 3 channels. Note that equilibrium queuing develops substantially earlier for a MTTS of 20 sec than for a MTTS of 50 sec. The stationary, expected queue length is also shown to be quite sensitive to the service rate.

Another description of the system which exhibits interesting transients is the probability that there are  $n$  customers in the system at time  $t$ . This is shown in Figure 4 for the case of 5 FOs and 3 channels with a MTTS of 50 sec. Also shown in this figure is the expected queue length. By comparing Figures 3 and 4, one can appreciate the sensitivity of  $E[Q]$  to the number of customers.



For a system having 5 FOs and 3 channels, the effect of reducing the MTTS to 20 sec is shown in Figures 5 and 6. The equilibrium probability that four customers are present is less than 0.02 whereas with a MTTS of 50 sec, this probability is 0.14 (from Figure 4).

For a battalion-level commo system with all nine FOs using the three FM channels with equal intensity, the stationary probability that an FO must wait for a channel and the expected queue length are shown in Figure 7 as functions of the mean time to serve. Clearly if such a system were to be feasible with the Cannon-Launched Guided Projectile System (CLGP), the MTTS would have to be very short -- less than 12 seconds to assure a 90% chance of capturing a channel promptly. If the channels were divided so that three FOs were compelled to share one channel, the resulting queue behavior would differ from the 9 FO - 3 channel case even tho the ratio of FOs to channels is identical. Results for the 3 FO - 1 channel case are shown in Figure 8. These results should be compared with those of Figure 7. Other things being equal, the segregation of channels is seen to increase the expected queue and probability of waiting. As shown in Figure 9, the expected time to wait,  $E[W]$ , and the expected time to wait, given a requirement to wait, are both strong functions of the mean time to serve for the case of 3 FOs and 1 channel. The equations used in developing the results for the case of 3 FOs and 1 channel are shown in the next section (p. 22).



RESULTS OF COMMO QUEUING  
MODEL -- SYSTEM OCCUPANCY  
AND EXPECTED QUEUE LENGTH

Parameters

510s, 3 Channels  
MTFA per PO 60 sec  
MTIS 50 sec

FIGURE 4

Probability of  $n$   
in the System

$n = 0$

$E[Q]$

Time, (sec)





FIGURE 5

PROBABILITY THAT THERE ARE  
N CUSTOMERS IN THE COMMO SYSTEM

Parameters  
5 FOS, 3 Channels  
MTBA per FO 60 sec  
MTTS 20 sec

Probability

$n = 0$

$n = 1$

$n = 2$

$n = 3$

$n = 4$

Time, (sec)

FIGURE 6

RESULTS OF COMMO QUEUING  
MODEL -- SYSTEM OCCUPANCY  
AND EXPECTED QUEUE LENGTH

Parameters  
5 FOS, 3 Channels  
MTBA per FO 60 sec  
MTTS 10 sec

Probability of n  
in the System

$E[q]$

$n = 4$

$n = 5$

Time, (sec)



STATIONARY PROBABILITY OF  
FO WAITING AND EXPECTED  
QUEUE LENGTH VERSUS MTTs

Parameters  
9 FOS, 3 Channels  
MTBA per FO 60 sec

FIGURE 7

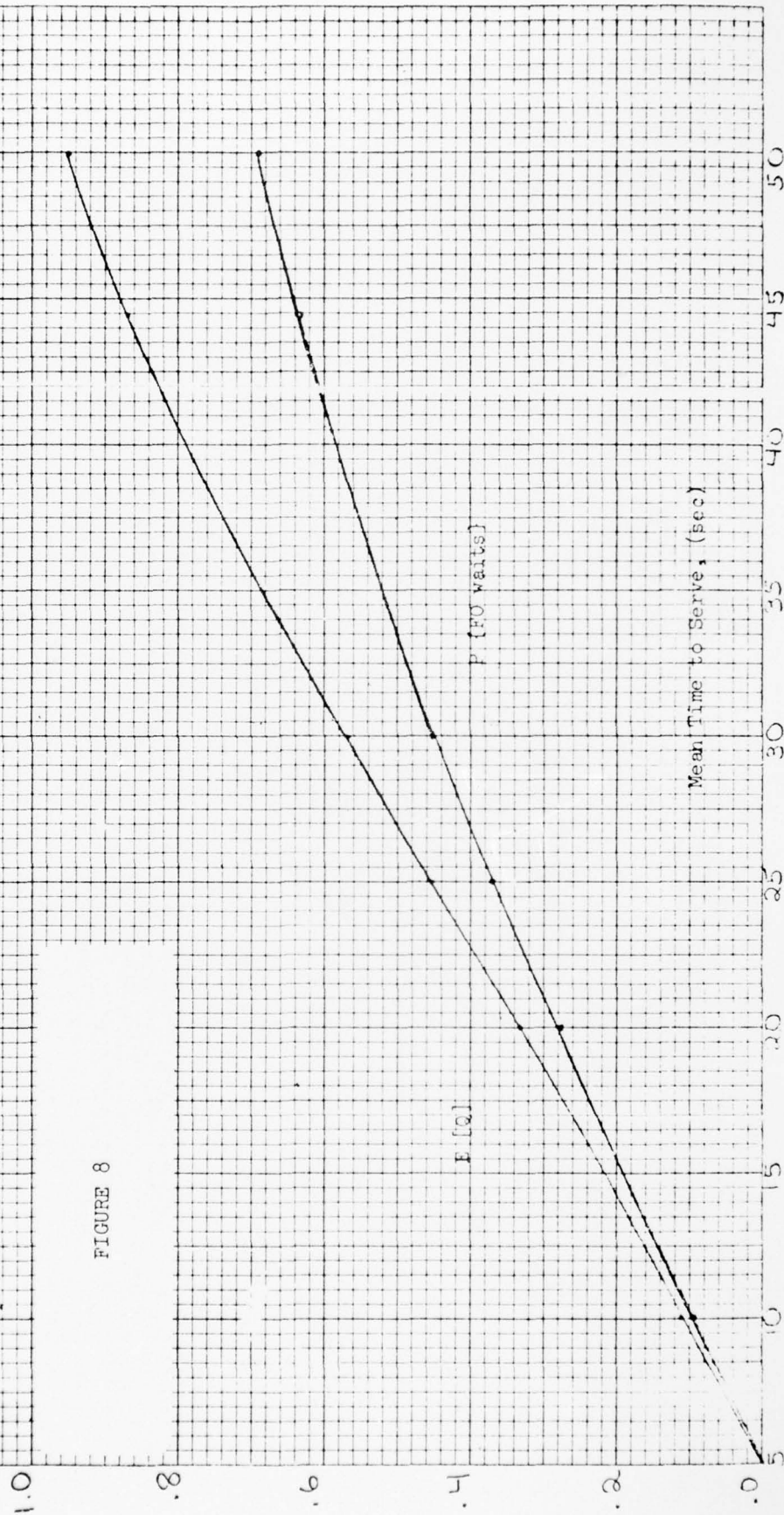


STATIONARY PROBABILITY OF  
FO WAITING AND EXPECTED  
QUEUE LENGTH VERSUS MTTs

Parameters

3 FOS, 1 Channel  
MTBA per FO, 60 sec

FIGURE 8



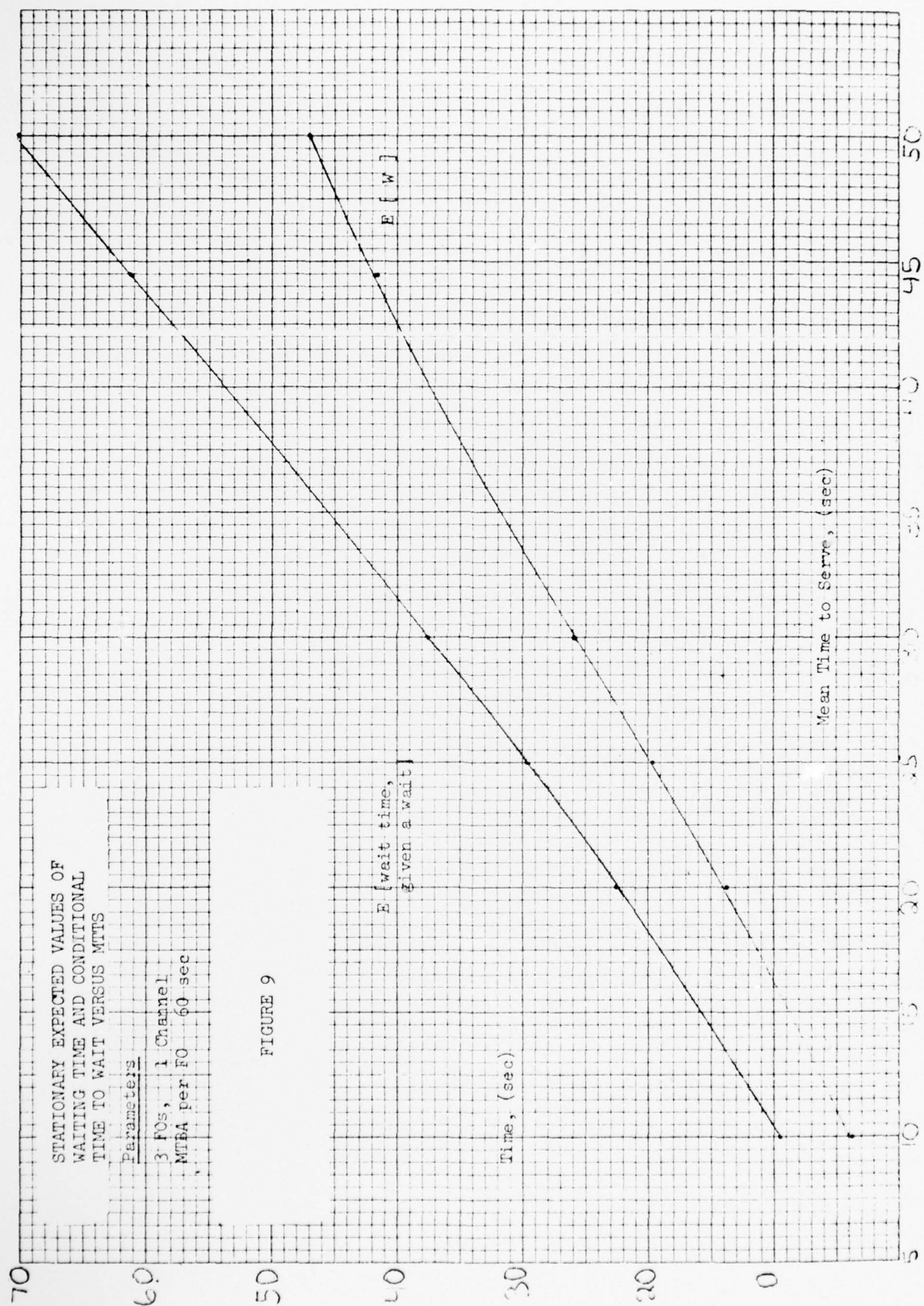


FIGURE 9



### Segregated Communications

In this section we consider the situation in which the conventional three FM channels per field artillery battalion are allocated among the FO users (customers) so that each channel is given to a separate set of users which pre-empt all others in using that channel. Assume these customers are CLGP FOs. In the case where 3 channels are shared by 9 FOs, segregation of the channels uniformly results in 3 FOs sharing a single channel.

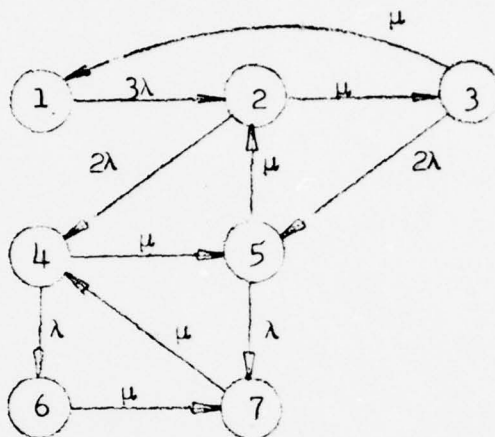
This situation would eliminate the need for a CLGP FO to change frequency and would permit convenient, direct communications between an FO and a particular battery. However the loss of flexibility created by this arrangement and the poorer service given FOs relative to an aggregated system having the same MTTs does not appear to justify the simplification.

The definition of states and the state diagram for this case is shown below in Figure 10.

Figure 10a.

State	Configuration			No. of FOs present
1	0	0	0	0
2	0	1	0	1
3	0	0	1	1
4	1	1	0	2
5	1	0	1	2
6	2	1	0	3
7	2	0	1	3

Figure 10b. State Transition Diagram



Using the state diagram and the methods outlined on p. 7 one can write the state equations for the case of single-channel pre-emptive queuing with 3 FOS. These are as follows.

State Equations for the Single-Channel Case

$$\dot{P}_1 = -3\lambda P_1 + \mu P_3$$

$$\dot{P}_2 = 3\lambda P_1 - (\mu + 2\lambda) P_2 + \mu P_5$$

$$\dot{P}_3 = \mu P_2 - (\mu + 2\lambda) P_3$$

$$\dot{P}_4 = 2\lambda P_2 - (\lambda + \mu) P_4 + \mu P_7$$

$$\dot{P}_5 = 2\lambda P_3 + \mu P_4 - (\lambda + \mu) P_5$$

$$\dot{P}_6 = \lambda P_4 - \mu P_6$$

$$\dot{P}_7 = \lambda P_5 + \mu P_6 - \mu P_7$$

(8)



States where CLGP FO is waiting: 4, 5, 6, 7

$$\begin{aligned} P\{\text{CLGP FO is waiting}\} &= P_4(t) + P_5(t) \\ &+ P_6(t) + P_7(t) \end{aligned} \quad (9)$$

Let  $E[X_k]$  be the expected time a CLGP FO must wait given the system is in the  $k$ th state when he arrives. Then

$$\begin{aligned} E[X_2] &= 2 \mu^{-1} \\ E[X_3] &= \mu^{-1} \\ E[X_4] &= 4 \mu^{-1} \\ E[X_5] &= 3 \mu^{-1} \end{aligned} \quad (10)$$

Thus the expected time to wait given requirement to wait is

$$\mu^{-1} [2 P_2 + P_3 + 4 P_4 + 3 P_5] / \sum_{k=2}^5 P_k \quad (11)$$

### Parameters

The state equations for this case (8) were solved for several values of  $\mu$  and with two functional forms for  $\lambda(t)$ . If one is concerned about the queuing of fire requests generated by outposted CLGP FOs, it is necessary to consider a functional form for  $\lambda(t)$  which is at least similar to what may be anticipated in a typical CLGP scenario. It is anticipated that in such a scenario, CLGP FOs would be present with a company-sized maneuver element used as a blocking or screening

force. In this instance the forces (and the accompanying FOs) do not intend to hold their position very long. For this reason the fire request rate which will build up gradually as targets begin to appear, will be expected to diminish rapidly at some point in time when the FOs are compelled to displace for their own security. For tank targets moving toward the outpost positions in assault without the benefit of improved roads and without encountering mines an average speed of advance of about 5 m/sec is expected. It is also expected that FOs would withdraw when advancing tanks come within about 1 km of their position. In this case outposted FOs will have about 10 minutes of activity before displacing.

The rate of fire requests at peak demand is limited by several considerations other than just the rate of appearance of targets. For example, the activities required of the FO party, viz., target detection and acquisition, FDC cuing, target tracking and laser designation also may limit the maximum value of  $\lambda$ . Even if one member of the FO party was committed to detection and acquisition of targets, it is doubtful whether the peak rate for fire requests would exceed 1/minute. On the basis of a peak request rate of  $1/60 \text{ (sec)}^{-1}$  achieved after 600 sec., the following arrival rate function was chosen.

$$\lambda = (1/60) \sin \frac{\pi t}{1200}, \quad (1.2)$$

for  $0 \leq t \leq 600 \text{ sec}$  and

$$\lambda = 0$$

for  $t > 600 \text{ sec.}$

In addition to  $\lambda(t)$  given by (12), the differential equations (8) were solved for

$$\begin{aligned}\lambda &= 1/60 & , & & 0 \leq t \leq 600 \text{ sec} \\ &= 0 & , & & t > 600 \text{ sec.}\end{aligned}\tag{13}$$

The initial conditions for all cases examined are

$$\begin{aligned}P_1(0) &= 1 \\ P_k(0) &= 0 & , & & k \neq 1\end{aligned}\tag{14}$$

### Results

Using  $\lambda(t)$  as in (12), the probability that a CLGP FO waits is shown in Figure 11 for several values of the MTTs. Figures 12 and 13 were developed using equation (13), with  $\lambda$  constant. Figure 12 displays the probability that a CLGP FO is waiting for a channel for the same set of values of MTTs used in Figure 11. Further, for other customers than CLGP FOs who may wish to use the commo channel, the probability of not being blocked is shown in Figure 13.

These results clearly indicate that even for a MTTs of 25 sec ( $\lambda = 0.08 \text{ sec}^{-1}$ ), a single commo channel is hopelessly inadequate to satisfy the demands of three CLGP FOs. Additionally, essentially no channel capacity remains for other than CLGP FOs.

FIGURE 11

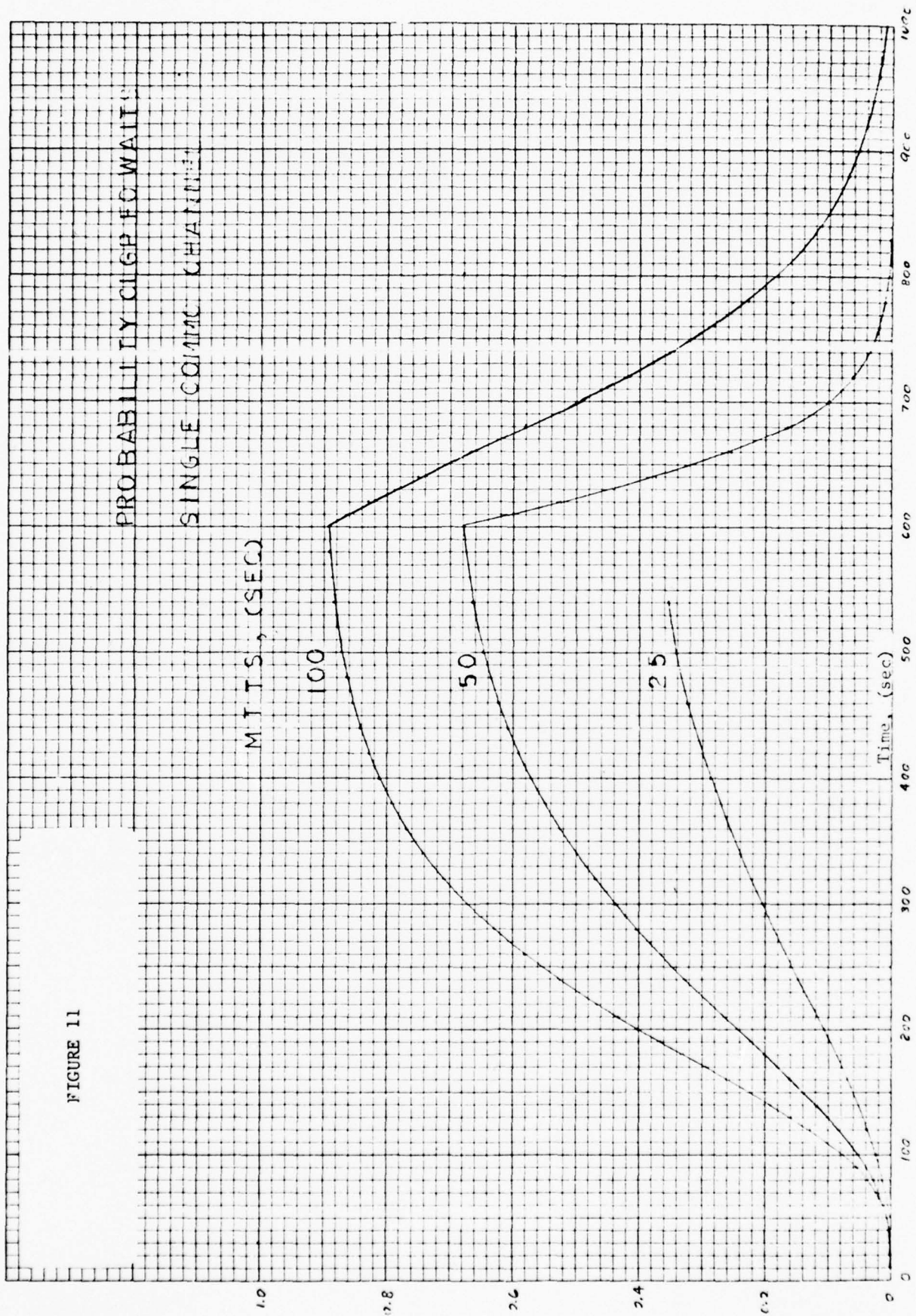




FIGURE 12

PROBABILITY CLIP-FO-WAITS  
SINGLE COMM CHANNEL

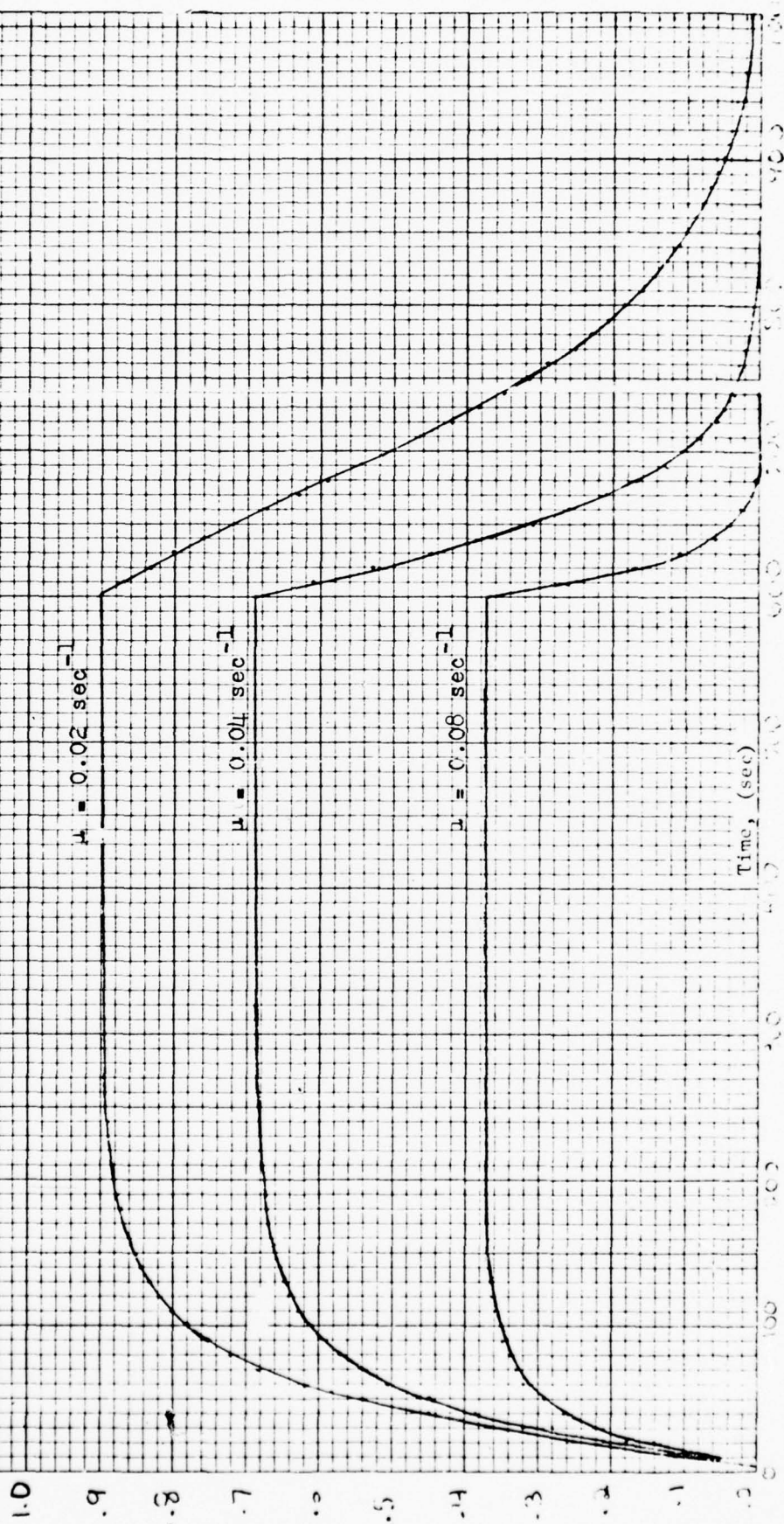
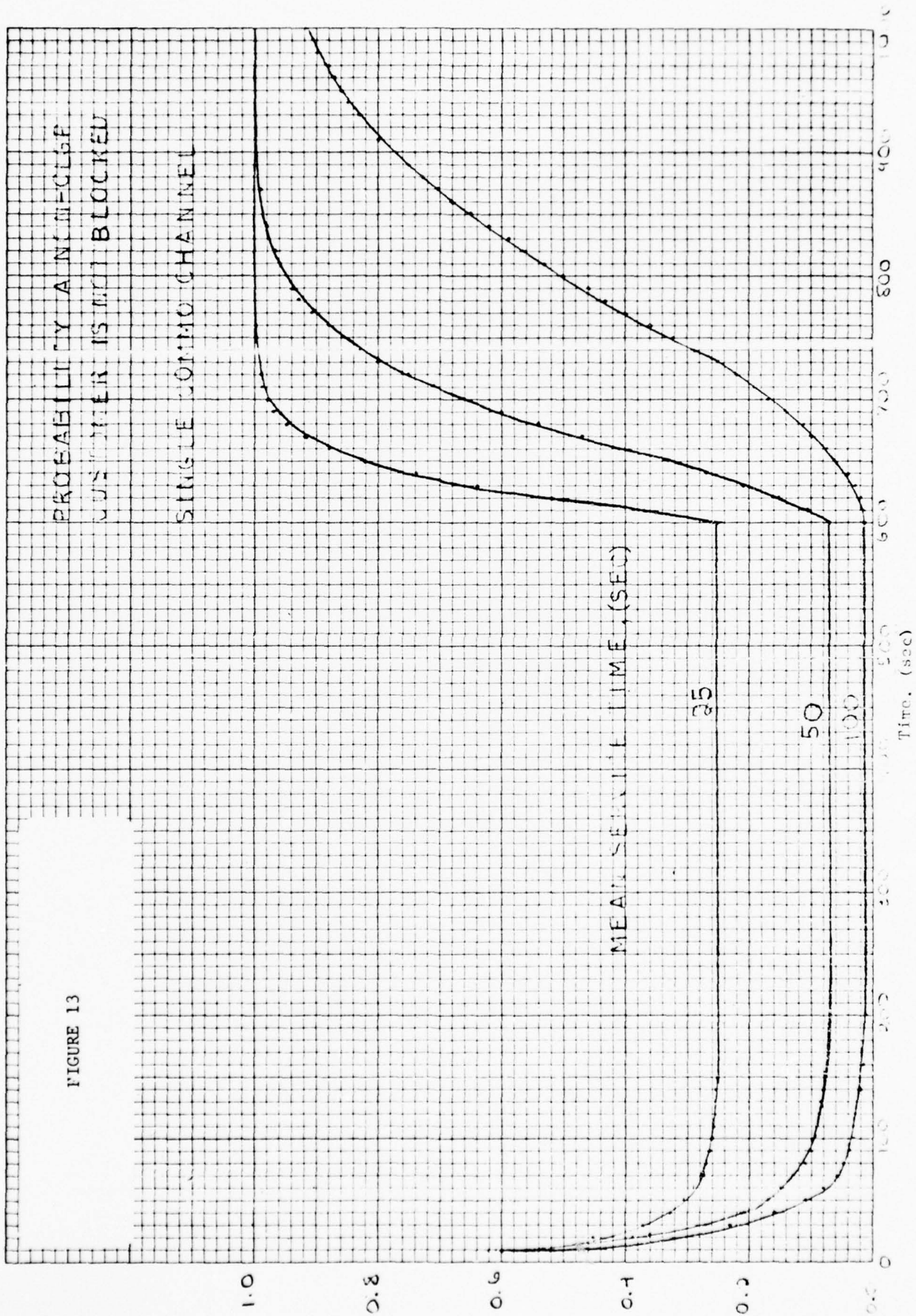




FIGURE 13



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## A Communications Simulation

A simulation model of an artillery commo system was developed to permit the examination of a variety of communications system configurations with greater ease and flexibility than that achieved thru analytic models. The results of this simulation were compared with analytic predictions for simple system configurations such as

- a. 1 channel, 3 FOs
- b. 2 channels, 3 FOs
- c. 3 channels, 5 FOs
- d. 3 channels, 9 FOs,

each configuration having two stages of service per channel and with only one type of message being transmitted.

The comparison facilitated testing and debugging the simulation. Aside from the expected stochastic fluctuations in simulation outputs, there is essentially perfect agreement between the simulation and analytical results.

### The FO Alert

To appropriately model the communications traffic for the Cannon Launched Guided Projectile System (CLGP), it is necessary to treat the messages transmitted to the FOs by the FDC alerting them to activate their lasers. It will be necessary to take special measures to insure that the alert (and its confirmation) get to the FO at the proper time. However, in the simulation, it is not assumed that a fire request requires a dedicated channel thru the alert.

For reasons of FO security it is desirable to minimize the designation interval. Generally this designation interval will be 10 to 15 seconds long as required by the optimal time for guidance. Consequently, a 5 second or greater lag in transmitting the alert will begin to substantially affect guidance accuracy.

To insure that the alert is punctual, the FDC must allocate a slack interval prior to the actual alert so as to insure a timely transmission. This slack period blocks incoming message requests from other FOs for the duration of the alert. The greater this slack period the greater is the confidence that the alert will be no greater than, say, 5 seconds late. Because the expense of a guidance failure is great, it is anticipated that a high probability of timely alert will be required. This will require a non-negligible alert interval which subtracts thereby from channel capacity. Clearly a tradeoff must be made between loss of channel capacity suffered during an alert and loss of guidance accuracy suffered when an alert is delayed. Of course, it is possible to increase channel capacity by formatting messages so as to require less time (and so as to permit interruption by the FDC). In the parlance of queuing theory, this is equivalent to the reduction of service time. With reduced service time a higher probability that the alert will be punctual is achieved. Another element in this tradeoff is the possible addition of another channel or dedication of another channel to CLGP FOs.

The simulation facilitates tradeoffs between system parameters. The manner in which the alert is treated is as follows. An alert event is scheduled to be sent

to a particular FO on the channel he used in his request. This alert arrives after a prescribed artillery system response time (60 sec) and lasts for a given alert interval. If the channel is busy at this time, the alert is delayed until that stage of service is completed. The alert interval was treated parametrically. Results were obtained for intervals of 5 and 10 seconds. These are shown in Table 1.

#### Queuing Simulation Results

Examples of queuing simulation results are shown in Figures 14 thru 16. Figure 14 shows the cumulative distribution function for the time interval over which the FDC is required to wait to transmit an alert. Figure 15 compares certain statistics from the analytical model, which does not treat an alert, with a simulation using the alert logic. Figure 16 compares the expected number of messages awaiting transmission for an aggregated commo system of 9 FOs and 3 channels with a MTTS of 50 sec with that for a MTTS of 20 sec. All of the simulation results were obtained for the case of constant arrival rate  $\lambda$ , starting with an empty system.



Table 1  
Table of Queuing Simulation Results for MTBA of 60 Seconds

System Description			Output Variables				
MTBS (sec)	No. Channels	No. FOS	Alert Interval (sec)	P{alert not blocked}	P{alert waits < 10 sec}	E{time alert waited} (sec)	E{no. in queue}
20	1	3	5	0.358	0.774	9.64	0.45
50	1	3	5	0.199	0.474	24.46	1.05
20	2	3	5	0.613	0.862	9.77	0.03
50	2	3	5	0.415	0.613	24.59	0.14
				P{alert waits < 15 sec}			
20	1	3	10	0.294	0.852	9.64	0.61
50	1	3	10	0.193	0.606	24.32	1.14
20	2	3	10	0.578	0.913	9.56	0.05
50	2	3	10	0.401	0.675	24.78	0.17



FIGURE 14

QUEUEING SIMULATION RESULTS --  
Probability FDC Must Wait at Least W to  
Transmit an Alert Given Requirement to  
Wait.

Parameters  
3 FOS, 1 Channel  
MUBA per FO 60 sec  
MINS per stage 10 sec

Alert Interval 5 sec

Probability

Wait Time, (sec)

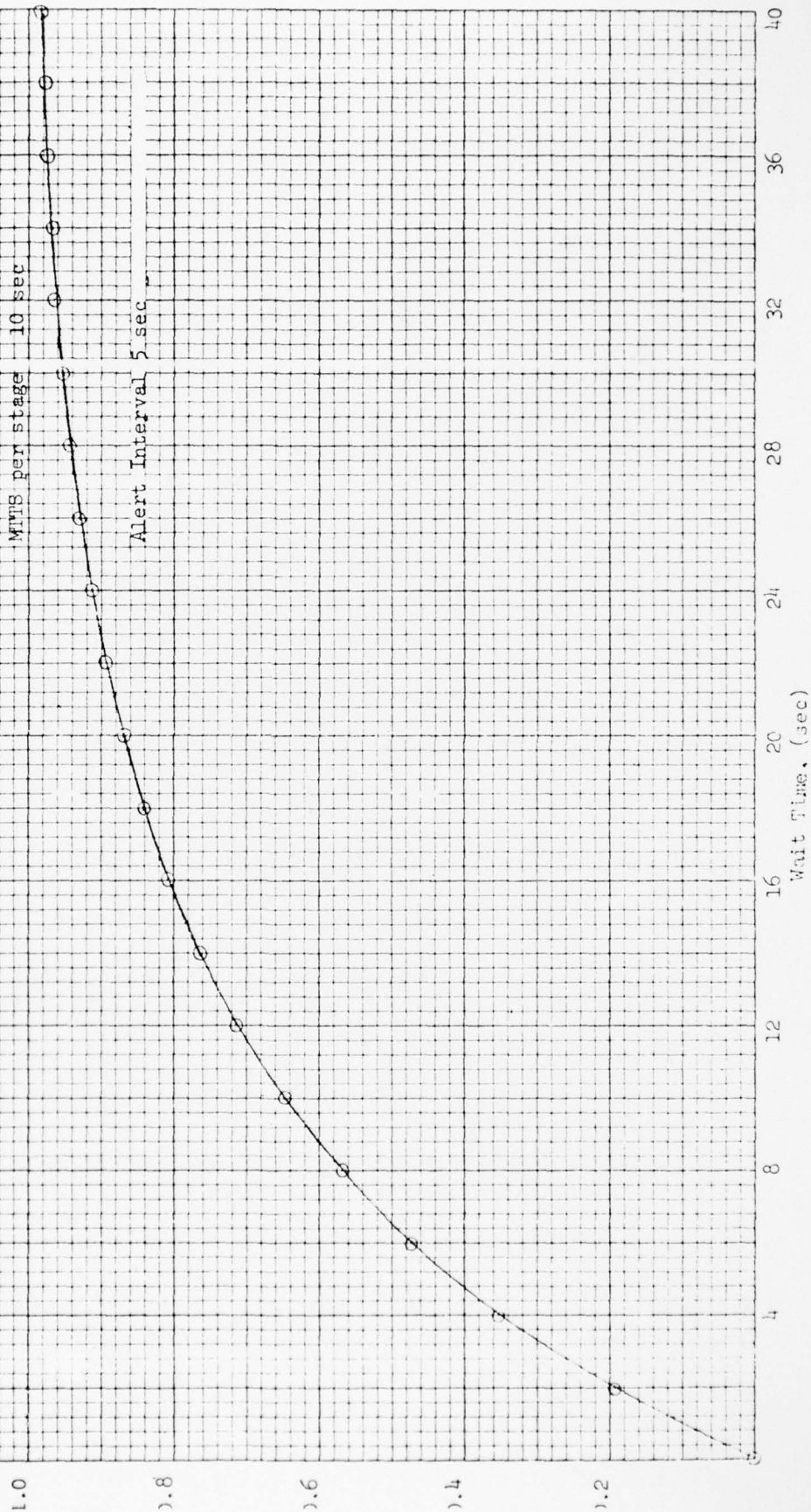


FIGURE 15

COMPARISON OF ANALYTIC COMMUNICATIONS  
MODEL WITH SIMULATION RESULTS

Simulation Treats 5 Second Alert  
Parameters

3 FOS, 1 Channel

Service Time 50. sec

Simulation Sample Size 2000

$P\{0 \text{ in Queue}\}$

Points were obtained  
from the simulation.

$E[Q]$

Time, (sec)

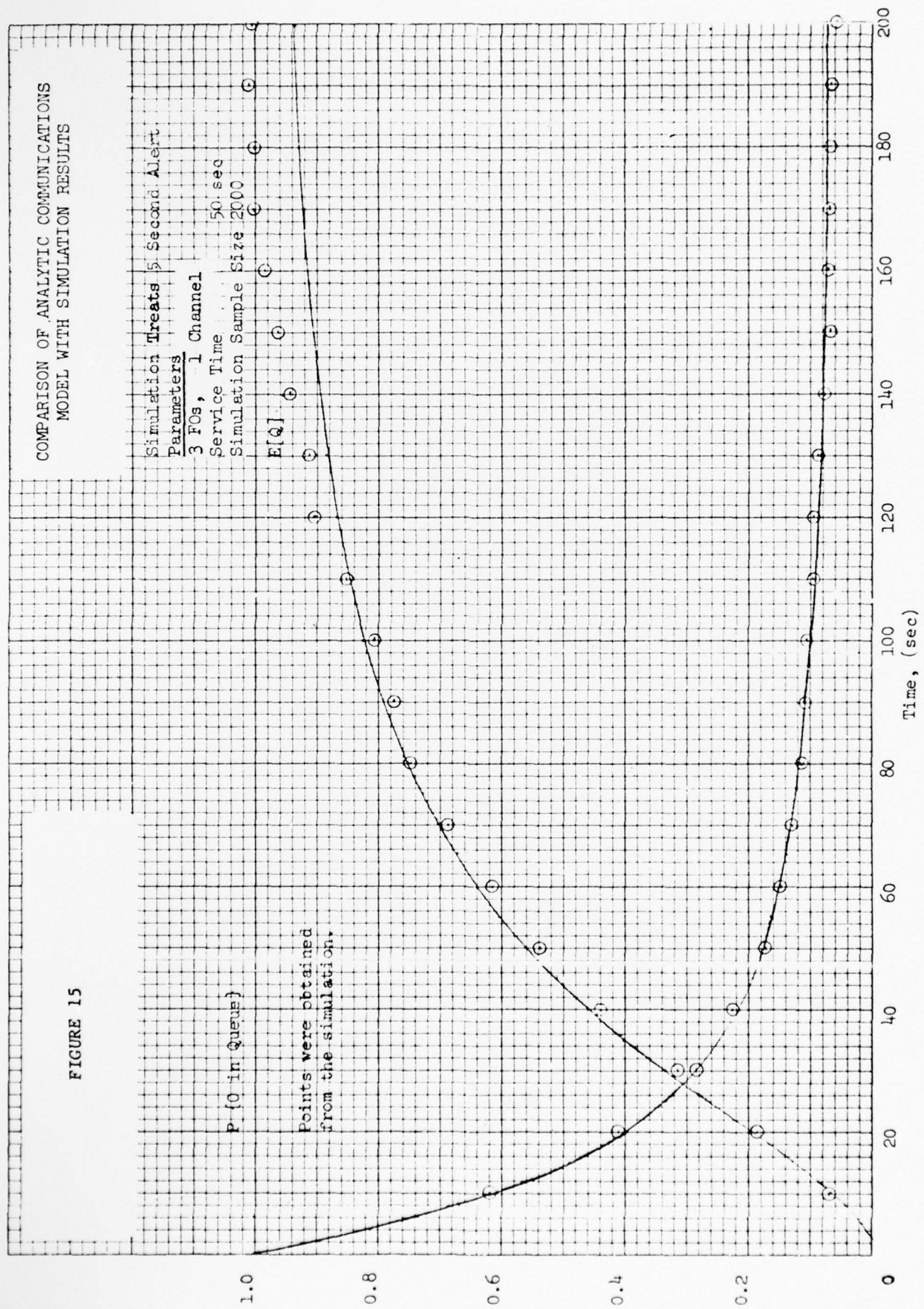
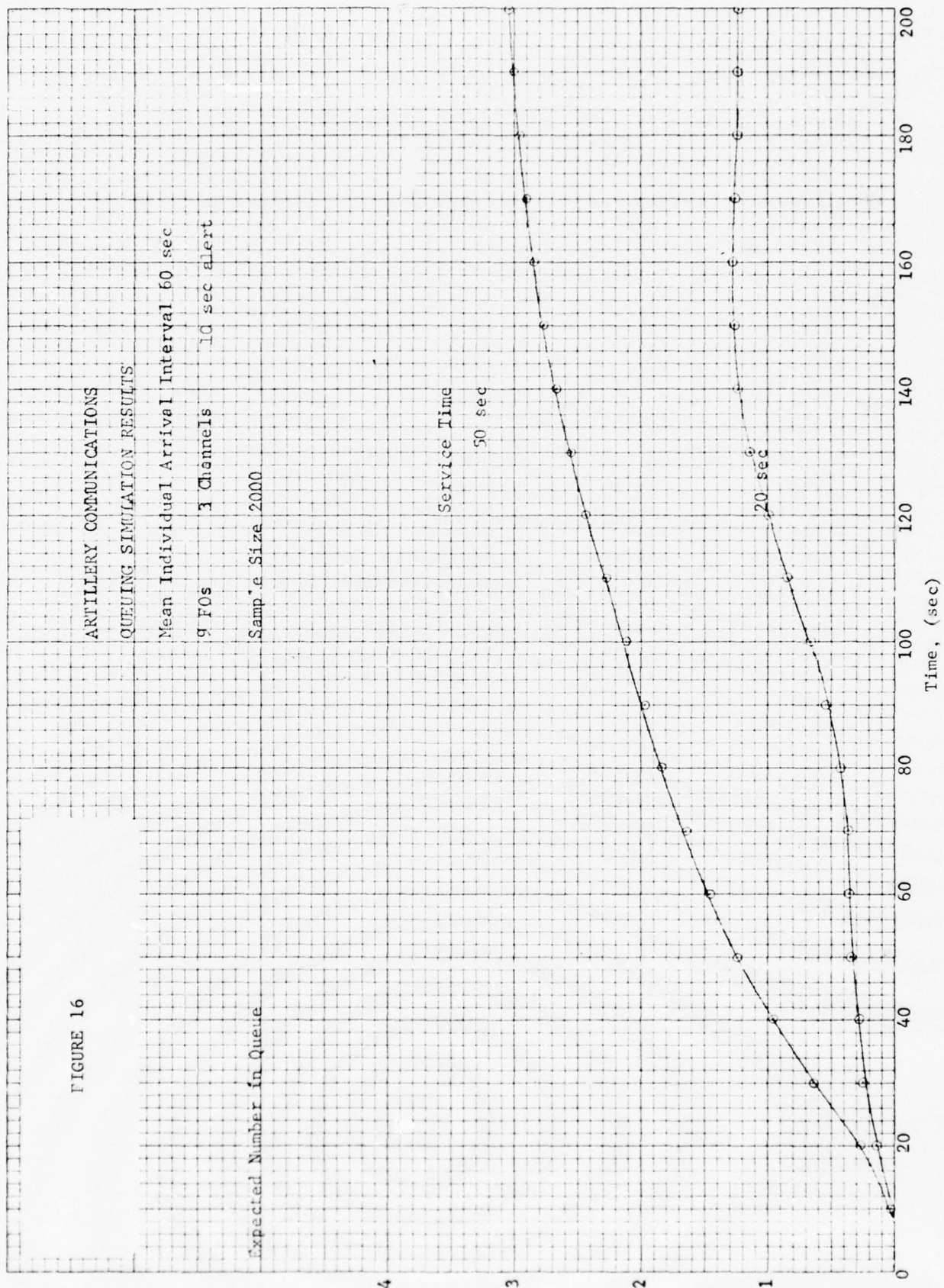


FIGURE 16





### Interpretation of the Simulation Results

In order to satisfactorily handle the expected message traffic from three CLGP FOs, it appears that two (or more) channels are required. A single channel would not be adequate even with the short MTTS of 20 sec. The probability that a timely alert occurs is unacceptably small for a single-channel system.

Even with a two-channel commo system, the probability of a timely alert is less than, say, 90% unless the MTTS is 20 sec or less and unless an adequate slack period or alert interval is allowed prior to the latest time at which the FO must turn on his laser. For example, from Table 1 we note that with a MTTS of 20 sec and a 10 sec slack interval, the probability of the FDC being no more than 5 sec late is about 91% even tho the expected number in the queue is only 0.05.



#### REFERENCES

1. Olson, Stuart W. The Numerical Solution of Transient Queuing Problems, Technical Report No. PAA-TRI-72, Systems Analysis Division, U.S. Army Weapons Command, Rock Island, Illinois, May 1972.
2. \_\_\_\_\_, "Field Artillery Reaction Times from Target Detection to Weapon Firing for Various Combinations of Cannon Support Systems," letter report dated December 1971.
3. Harper, Donald G., et al. Communications Traffic Analysis of the Fire Support System of Command and Control Information System, CDOG Project 65-2, Final Report, University of Oklahoma Research Institute, February 1966.

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DEPARTMENT OF THE ARMY MATERIEL READINESS  
HEADQUARTERS, UNITED STATES ARMY ARMAMENT/COMMAND  
ROCK ISLAND, ILLINOIS 61201

REPLY TO  
ATTENTION OF:

25 APR 1977

DRSAR-SA

SUBJECT: Computer Simulations for the Ballistics of Copperhead

Director  
Ballistics Research Laboratory  
ATTN: DRDAR-BLL-FT (Mr. D. Clark)  
Aberdeen Proving Ground, MD 21005

1. References:

a. MFR, DRSAR-SAM, 23 Jan 76, subject: Description of Computer Program: Exterior Ballistics of Boosted Rockets. This MFR appears in the Systems Analysis Directorate Activities Summary January - February 1976, DRSAR/SA/N-42.

b. Letter, SARRI-LR to AMXBR-EB-FT, 8 Jan 76, subject: CLGP Terminal Homing Models.

c. MFR, DRSAR-SAM, 20 Apr 77, subject: An Improved Algorithm for the Glide Mode of Copperhead Used in a 3 DOF Flight Simulation (Incl 1).

2. The modified three-degree-of-freedom (3 DOF) ballistic simulation described in Ref 1a was sent to BRL in January 1976. The computer program was intended to be an efficient means for generating range tables for Copperhead, including the glide (FUFO) mode. In achieving this objective a very simple algorithm--glide-hold, discussed in Ref 1a--was used to simulate the FUFO mode of flight.

3. As delineated in Ref 1b, good agreement was obtained between ranges generated by the ARMCOM Systems Analysis Model (SA 3) and the elaborate Rodman six-degree-of-freedom model (ROD 6). However, it is believed that the agreement for glide trajectories between SA 3 and ROD 6 is somewhat misleading since all glide-mode comparisons were made for firings in the lower register and for shots in which the glide angle did not change greatly from the nominal, twenty-degree value.

4. In order to improve agreement between these models for off-nominal conditions using the glide mode, SA 3 has been changed to mechanize a new glide algorithm. This algorithm is described in Ref 1c (Incl 1)

4 5 APR 1977

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SUBJECT: Computer Simulations for the Ballistics of Copperhead

where results are compared with results of 6 DOF models used by Martin Marietta Corporation (MMC 6) and by Rodman Labs (ROD 6).

5. Our computational experience with this version of SA 3 (April 1977) indicates that trajectory shape, particularly glide angle, better represents results of 6 DOF models than the 1976 version. To expedite your receipt of this model, the source program card deck with listing is being sent under separate cover.

FOR THE COMMANDER:

SIGNED

1 Incl  
as

OTTO F. HAASE, JR.  
Acting Director  
Systems Analysis Directorate

CF:

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Mr. E. Manley/Mr. E. Zimpo/COL R. Nulk, Dover, NJ 07801

Project Manager, Cannon Artillery Weapons Systems, ATTN: DRCPM-CAWS-SIA/

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Commander, White Sands Missile Range, ATTN: STEWS-TE-PC/Mr. S. Kadner,

Bldg 1544, White Sands, NM 88002

Commander, US Army Armament Materiel Readiness Command, ATTN: DRDAR-LCA-PP/

Dr. Amoruso, Rock Island, IL 61201

20 Apr 77

## MEMORANDUM FOR RECORD

SUBJECT: An Improved Algorithm for the Glide Mode of Copperhead Used in a 3 DOF Flight Simulation

## 1. References:

a. MFR, DRSAR-SAM, 23 Jan 76, subject: Description of Computer Program: Exterior Ballistics of Boosted Rockets. This MFR appears in the Systems Analysis Directorate Activities Summary January - February 1976, DRSAR/SA/N-42.

b. Letter, SARRI-LR to AMXBR-EB-FT, 8 Jan 76, subject: CLGP Terminal Homing Models.

2. Previous Work.

In modeling the glide, FUF0, or attitude-hold mode of Copperhead within the general purpose, point-mass, three-degree-of-freedom (3 DOF) program EXBAL, one generally desires computational efficiency and fidelity of pertinent flight variables to the values obtained from more elaborate six-degree-of-freedom (6 DOF) models. In Ref 1a an algorithm is presented in which the glide angle is fixed or held by appropriate choice of aerodynamic forces. Two options are permitted. In one, the glide angle is held after a prescribed glide angle is reached. In the other option the glide angle achieved after a prescribed time, TENABL, is held for the remainder of the flight.

3. Altho the glide mode of Copperhead does not actually operate in this manner, Ref 1b shows that range estimates of good accuracy can be obtained using this algorithm. However, several shortcomings of this algorithm have been observed. For one, the glide angle actually assumed in the glide mode is not constant but takes on a value such as to preserve body attitude. Parenthetically, note that attitude-hold is most descriptive of this mode of flight. Thus, the glide angle is only approximately constant and may depart significantly from the nominal -20 degrees desired. For these trajectories, a glide-hold algorithm does not adequately represent trajectory shape.

4. Another shortcoming of the earlier (1976) glide algorithm is the absence of a direct input of the Copperhead time setting used by the 6 DOF models,



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a 3 DOF Flight Simulation

$T_0$ . This timer setting represents the time from launch at which the 30 v main projectile power becomes available. The time  $T_0$  is the (integer) time (in seconds) set by code switch into the projectile plus the approximate 0.5 second interval to account for the following lags not included in the projectile timer: (a) the approximately 0.30 second interval for the 11 v battery to activate following launch and the approximately 0.20 second interval required for the 30 v battery to achieve acceptable power after it is activated. All other events occurring within the projectile are sequenced from the base time  $T_0$ . These events are tabulated below:

SEQUENCE OF EVENTS IN COPPERHEAD TIMER

Event No.	Time (s)	Description
1.	$T_0$	timer sequence starts (main power at $\pm 30$ v)
2.	$T_0+2$	roll control starts
3.	$T_0+3$	initiate seeker gyro spinup
4.	$T_0+4$	attitude-hold enable switch is closed
5.	$T_0+5$	start to extend wings, apply g bias, and free gyro

5. Altho internal degrees of freedom within the projectile are not simulated with the 3 DOF model, it is important to properly incorporate their kinematic effect. In contrast to the earlier algorithm, one option of the improved algorithm described below treats the events which occur after  $T_0$  in a manner similar to that of the actual projectile. As with the previous algorithm, the principal assumption employed is that the controlled projectile always remains in a trim condition.

6. Options for the Improved Program.

There are three switchable options associated with the glide mode in the current version of EXBAL. All options require the glide mode switch IGLIDE to be set to unity. In option 1 a desired glide angle (GLIDE) is provided and the program calculates the body angle of attack required to

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hold body attitude after reaching the desired glide angle. Before attitude hold of the body is initiated an initial angle of attack is calculated such that the lift force equals the gravitational force normal to the velocity vector at that point in the flight. See Attachment 1 for details. This option may be useful when timer settings are unavailable.

7. In option 2 a time TENABL (in secs) is provided and CLIDE is set to -90 degrees. With this option the calculations for angle of attack to hold body attitude start at TENABL. A value of TENABL equal to that at which the velocity angle  $\theta = \text{CLIDE}$  produces a trajectory identical to that of option 1.

8. Option 3 requires the program user to provide time  $T_0$ , called TMOMMC in the program. This option is exercised by setting the switch MMCSW to unity. So far as projectile kinematics are concerned, the projectile remains ballistic until event number 4. When event number 4 occurs at  $T_0+4(s)$ , a control surface deflection in pitch,  $\delta_p$ , is calculated continuously and applied thruout the interval between events 4 and 5. This algorithm bases the calculation of  $\delta_p$  upon the equilibrium behavior of the projectile and autopilot between events 4 and 5. This behavior is approximately one in which the pitch deflection of the fins and the associated trim angle of attack are proportional to the average turning rate of the velocity vector,  $\bar{\dot{\theta}}$ . Thus,

$$\delta_p = K \bar{\dot{\theta}} \quad (K, \text{ a constant}).$$

K is approx. 0.5 deg/(deg/sec). The turning rate  $\dot{\theta}$  is calculated continuously by

$$\dot{\theta} = (\dot{x} \ddot{y} - \dot{y} \ddot{x}) / v^2.$$

An average value of  $\dot{\theta}$  is calculated by exponential smoothing. This, of course, assumes that the autopilot and projectile system exhibit approximately first-order dynamics between the events 4 and 5. Specifically,

$$\bar{\dot{\theta}} = \frac{\dot{\theta}}{1 + \tau s},$$

with  $\tau$  a time constant and  $s$  the Laplace differential operator. The value of  $\tau$  is approximately 0.2 secs. Using exponential smoothing, this equation is solved implicitly by the recursive procedure:

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$$\bar{\theta}_i = c_o \bar{\theta}_{i-1} + c_1 \dot{\theta}_i$$

with

$$c_o = e^{-h/\tau}$$

$$c_1 = 1 - c_o ,$$

where h is the integration time step, ie,

$$t_i = t_{i-1} + h .$$

Having calculated  $\delta_p$ , the trim angle of attack ( $\alpha_t$ ) is obtained by linear interpolation in the following table.

TRIM ATTACK\* VERSUS MACH NUMBER AND CONTROL DEFLECTION

Entries are  $\alpha_t$  (deg)

Mach Number	$\delta$ (deg)			
	0	5	10	15
0.5	0	7.4	12.1	17.7
0.8	0	7.2	11.8	17.2
0.9	0	7.4	11.6	17.1
1.0	0	7.1	11.3	17.1

\* Reference: Wind Tunnel Data Analysis of 3/4 Scale Model for the  
XM712 Projectile, 27 Feb 76, pp. 72-75.

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9. With all program options the normal body force is calculated from the normal force coefficient at trim. This coefficient is a function of Mach number,  $M$ , and trim attack,  $\alpha_t$ , ie,

$$C_N = C_N(M, \alpha_t) .$$

Values of  $C_N$  are obtained by two-way linear interpolation in a table as explained in Attachment 1.

10. Comparison of EXBAL Results With 6 DOF Models.

Several different Copperhead trajectories have been generated using the improved glide-mode algorithm in SA 3 for the purpose of comparing these results with those of the 6 DOF models. Results of three glide trajectories and a ballistic trajectory are shown in Table 3. Comparable results for the Rodman (ROD 6) and Martin Marietta Corporation (MMC) 6 DOF models are shown, where available. For all glide trajectories with SA 3, option number 3 was used. The ballistic option is compared with results of ROD 6 to demonstrate the virtual equivalence of program results when using this option.

11. Altho the range generated by SA 3 is quite comparable with that of ROD 6 and MMC in glide, it is noted that the final flight speed is about 1 to 2 m/s less than the 6 DOF results and the final glide velocity angle is about 0.1 to 0.3 degree less. For the 66.5 deg QE shot a trajectory is displayed in Figure 1. The range given by SA 3 is somewhat longer and the apogee higher than that of ROD 6. This in part, accounts for the 0.4 sec longer time of flight for SA 3.

12. A significant flight variable for the glide mode is the glide angle,  $\theta$ . This is shown versus time for both SA 3 and ROD 6 in Figure 2. During the transition interval from ballistic flight to attitude-hold flight--44.5 (s) to about 46 (s)--there is excellent agreement between these models. At the end of the flight the SA 3 result becomes about 0.3 degree less (steeper).

13. Conclusions.

In view of the excellent agreement displayed here, one is led to believe that the improved algorithm for the glide mode will prove suitable for range table generation. Further, the present mechanization does not require substantially more CPU time than the earlier (1976) version.



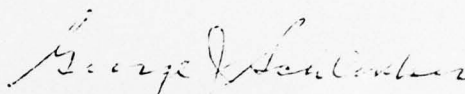
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SUBJECT: An Improved Algorithm for the Glide Mode of Copperhead Used in  
a 3 DOF Flight Simulation

In fact this program is still one-to-two orders of magnitude faster than the 6 DOF models.. This suggests that it could become the basis for a real-time F/C algorithm.

1 Incl  
• Attachment 1



GEORGE J. SCHLENKER  
Operations Research Analyst  
Methodology Division  
Systems Analysis Directorate

TABLE 3

## COMPARISONS OF RESULTS FROM COPPERHEAD SIMULATIONS

April 1977

Model	Inputs				Outputs				
	Mode	V <sub>o</sub> (f/s)	QE (deg)	T <sub>o</sub> (sec)	TOF (s)	Range (m)	Impact Speed (m/s)	Angle of Velocity (deg)	Body Angle (deg)
MMC	G	1290	66.5	40.5					
Rodman					61.63	7417	269.2	-56.58	-54.53
SA-3					62.05	7471	269.2	-56.63	-54.57
MMC		1663	28.5	20.5	44.07	12019	237.4	-19.50	-15.20
Rodman					44.01	11976	237.5	-19.48	-15.19
SA-3					44.38	12064	236.2	-19.61	-15.31
MMC		1918	43.5	28.5	80.59	19128	226.8	-20.13	-15.58
Rodman					80.53	19071	227.2	-20.34	-15.60
SA-3					80.97	19101	224.5	-20.53	-15.70
MMC	B	1290	22.5	9.5					
Rodman					26.42	7128	249.4	-29.29	-29.29
SA-3					26.45	7142	249.4	-29.33	-29.33

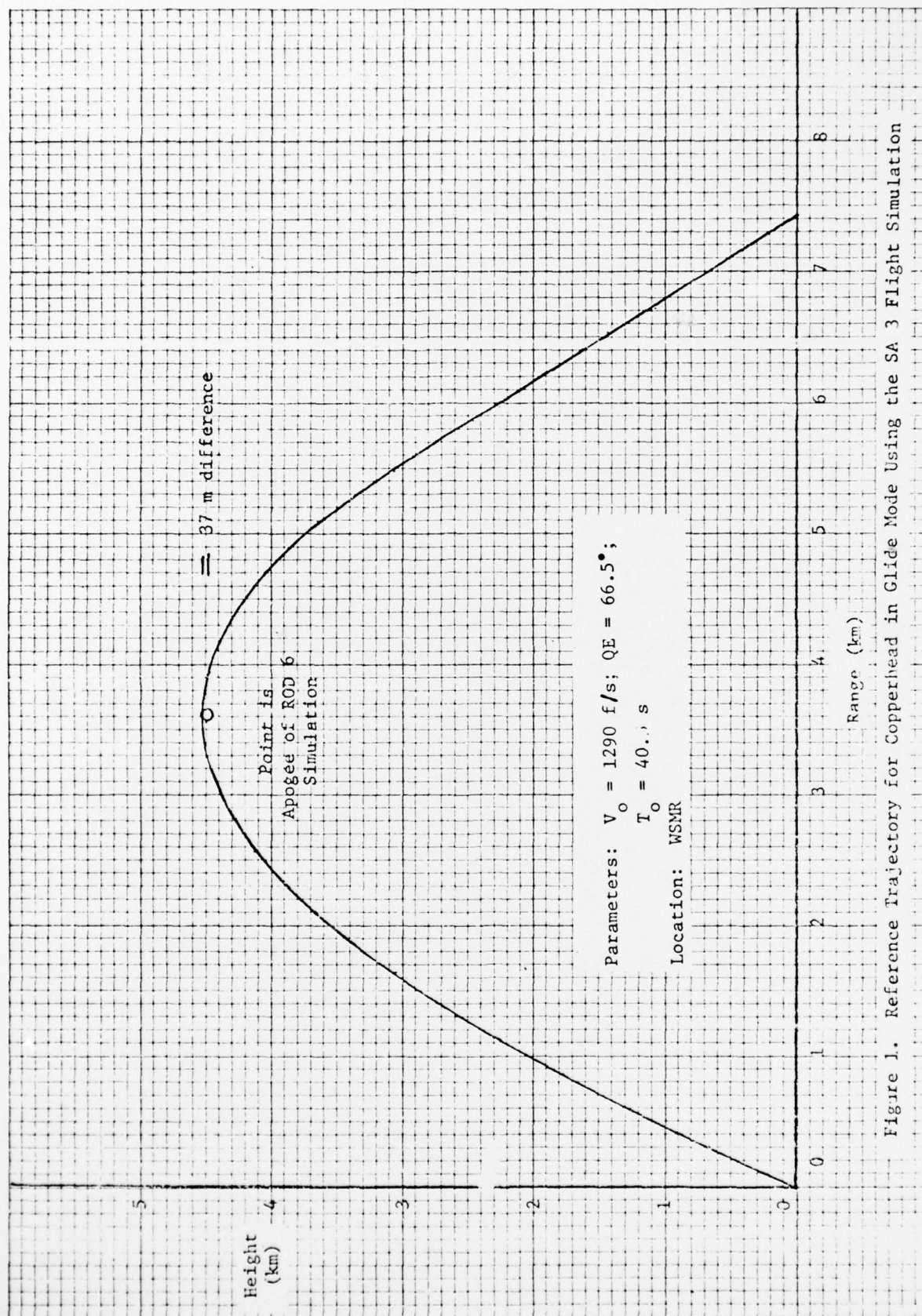


Figure 1. Reference Trajectory for Copperhead in Glide Mode Using the SA 3 Flight Simulation

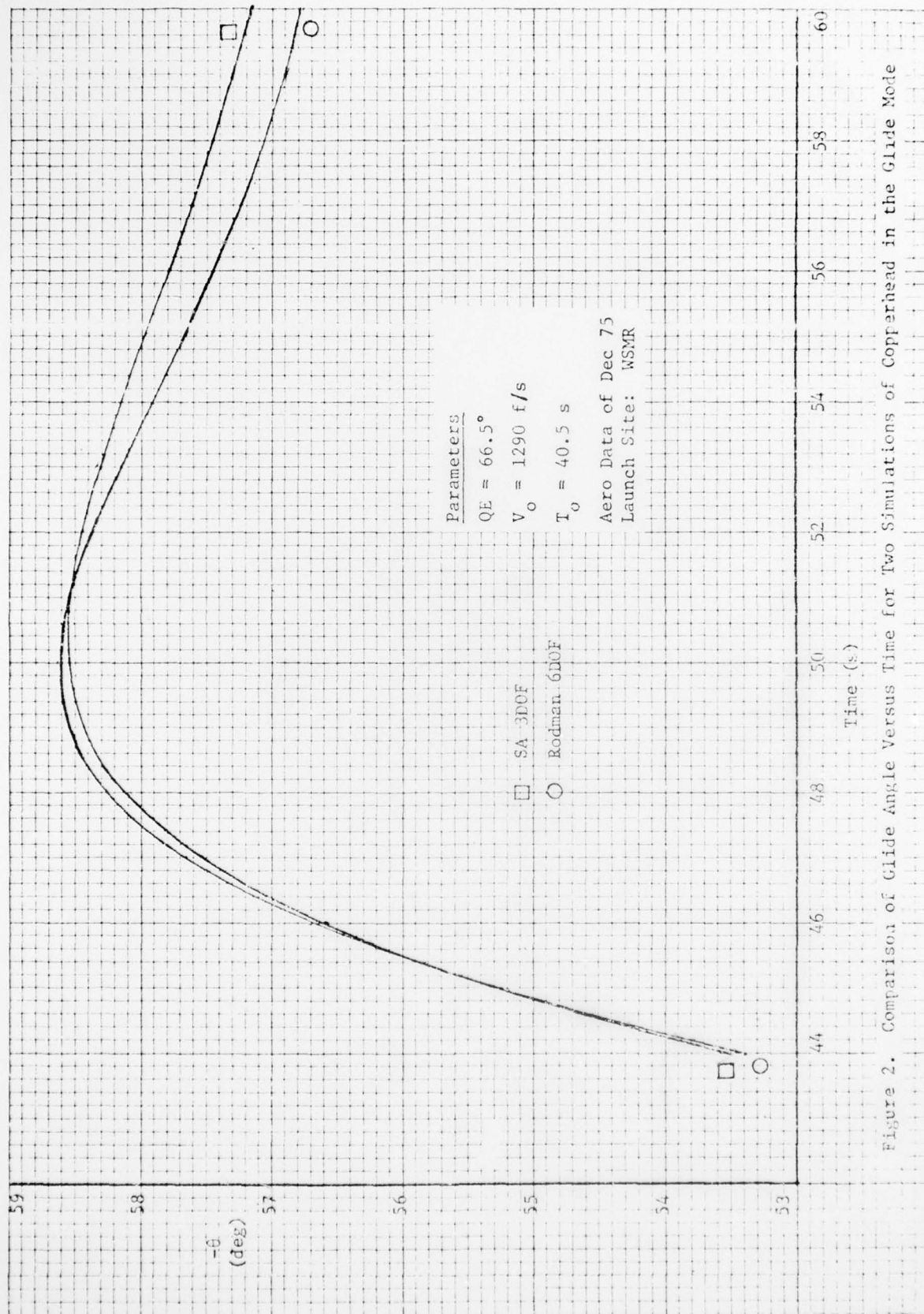


Figure 2. Comparison of Glide Angle Versus Time for Two Simulations of Copperhead in the Glide Mode



# ATTACHMENT 1

## Algorithm for Attitude-Hold Logic in Copperhead 3DOF Simulation

For simulations in which a preset-time option is not used, the program determines the time for commencement of attitude hold. Initially the glide angle  $\theta$  will approach the desired glide angle  $\theta_0$  algebraically from above. ( $\theta_0$  is negative) When  $\theta - \theta_0 \leq \epsilon$  ( $\epsilon$  positive), an initial angle of attack,  $\alpha_0$ , is computed such that the associated lift component equals the component of gravity normal to the velocity vector, ie, such that

$$F_L = M_p g \cos \theta$$

with

$$F_L = F_N \cos \alpha_0$$

and

$$F_N = C_N(0) Aq$$

with projectile mass  $M_p$  and reference area  $A$  and dynamic pressure  $q$ .

Since  $\alpha_0$  is small--typically less than  $10^\circ$ --an iterative procedure is employed in which the first iteration assumes that  $F_L = F_N$  so that

$$C_N^{(1)}(0) = \frac{M_p g \cos \theta}{Aq}$$

The value of  $\alpha_0^{(1)}$  is obtained by interpolation in the tabular function  $C_N(M, \alpha_t)$  with  $\alpha_t$  the trim angle of attack.

Thus,

$$C_N^{(1)}(0) = C_N(M^*, \alpha_0^{(1)}),$$

where  $M^*$  is the local Mach number. The interpolation procedure is described below.

Then, form the second iterate for the normal force coefficient:

$$C_N^{(2)}(0) = \frac{M_p g \cos \theta}{Aq \cos \alpha_0^{(1)}}$$

The value of  $\alpha_0$  obtained by requiring that  $C_N = C_N^{(2)}(0)$  is taken as the initial trim angle of attack at the start of attitude hold.

#### Interpolation Procedure

To calculate  $\alpha_0$ , interpolate on Mach number in the  $C_N(M, \alpha_t)$  table obtaining  $C_N(M^*, \alpha_t)$  at the local Mach number  $M^*$  for values of  $\alpha_t = \{0, 5, 10, 15 \text{ (deg)}\}$ . Then,  $\alpha_0$  is obtained by linearly interpolating with  $C_N(0)$  as argument in this table.

The value of initial body attitude is given by

$$\theta_b(0) = \theta + \alpha_0.$$

For subsequent calculations during the attitude-hold trajectory, the value of angle of attack,  $\alpha$ , is calculated which preserves body attitude, ie, for which  $\theta_b = \theta_b(0)$ .

Thus,

$$\alpha = \theta_b(0) - \theta.$$

If the above value of  $\alpha$  satisfies

$\alpha < \alpha_{tm}(M)$ , the maximum trim angle at  $M$  the instantaneous value of Mach number, the lift force is computed as

$$F_L = C_N(\alpha, M) A q \cos \alpha$$

with  $C_N$  obtained by two-way interpolation. Otherwise,  $\alpha$  is limited to  $\alpha_{tm}(M)$  and lift is calculated as

$$F_L = C_N(\alpha_{tm}, M) A q \cos \alpha_{tm}.$$

Induced drag due to lift is calculated as

$$F_{DI} = - C_N(\alpha, M) A q \sin \alpha.$$

TABLE 1. MAXIMUM TRIM ANGLE OF ATTACK  
VERSUS MACH NUMBER (AT  $\delta = 12^\circ$ )

Mach Number	$\alpha_t$ (max) (deg)
0.5	14.3
0.8	14.0
0.9	13.8
1.0	13.6

TABLE 2. NORMAL FORCE COEFFICIENT  
VERSUS MACH NUMBER AT TRIM ATTACK  
 $C_N(M, \alpha_t)$

Mach Number	$\alpha_t$ (deg)			
	0	5	10	15
0.5	0	1.1	2.1	3.1
0.8	0	1.2	2.1	2.9
0.9	0	1.3	2.2	3.0
1.0	0	1.3	2.5	3.7

## Quality Levels and Effectiveness

To inform the CG, ARRCOM, of examples of commodity requirements which, in analysis, provide only marginal benefit in operational cost-effectiveness at the force level.

1. Reference Project List Review, MG Eicher and Mr. Rhian, DRSAR-SA, 28 March 1977; question on Quality Control.
2. We have evaluated the influences of system alternatives on effectiveness achieved, mix of ammunition expended, and its dollar cost. We used our BLUE artillery force to engage total target arrays subject to TRADOC-approved mission scenarios, tactics, and usage doctrine. Results showed that some specified commodity requirements (performance attributes/improvements) were not justified by the realization of a significant improvement in operational cost-effectiveness of the field artillery. Examples (paras 3, 4 and 5) are summarized below.
3. The 155mm, HE, M549 RAP production rounds exhibited a greater range probable error (0.4% of range) than the developmental rounds (0.3% of range). Analysis showed that more precise (developmental) range variation significantly improved effectiveness only against small area targets engaged by observer-adjusted fire. On the basis of the Legal Mix target arrays, these engagement conditions were predicted to occur in only 15 percent of the M549 RAP firings. In the remaining 85 percent of the firings, when engaging larger targets and/or using unadjusted fire, no significant improvement was expected when using the developmental variation. This analysis was reported in a briefing to the CG, ARMCOM, in April 1974.
4. The improved precision specified for two time fuzes (the XM724 ET and the M577 MT), which are candidate fuzes for the second generation dual-purpose ICM's, did not result in a significant improvement in artillery force level operational effectiveness nor in a significant ammunition cost savings when compared with an alternative in which the ICM dispersion was based on the use of the less precise M565 MT fuze. This analysis was published in report DRSAR/SA/R-20, July 1976.
5. The 155mm, HE, XM708 hi-frag projectile provides significantly improved single round effectiveness on personnel targets in comparison to the 155mm, HE, M107 projectile. However, analysis showed that the XM708 yielded no significant effectiveness or cost advantage when included in the ammunition mix at the artillery force level (i.e., in comparison to the results with the M107 in the mix). This analysis was published in technical note DRSAR/SA/N-49, December 1976.
6. The 155mm, HE, M483A1 ICM (dual-purpose, second generation improved conventional munition) is an example of a commodity improvement which, in analysis, does significantly benefit operational cost-effectiveness at the force level. An analysis showed that the M483A1 provides essential anti-armor capability among the conventional, unguided 155mm field artillery munitions. Another analysis showed that only one-eighth as many battery volleys (fewer for some conditions) of M483A1's are required to achieve the same level of effectiveness against area personnel targets as that achieved by battery fire of 155mm, HE, M107 projectiles. The analysis pertinent to anti-armor capability was documented in a DRSAR-SAA Memorandum, November 1976. The other analysis was documented as a AMSAR-SAA Memorandum, dated April 1975.



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SUBJECT: Quality Levels and Effectiveness

7. The failure of one or more specific requirements (attributes or quality levels) of a commodity to be justified in terms of improved operational cost-effectiveness does not necessarily imply that the commodity should not be acquired. For instance, the justification for continued development of the XM724 electronic time fuze as a candidate fuze for the new ICM's (re: paragraph 4 above) was based on safety and the general evaluation of the electronic timing industry

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Director, Systems Analysis Directorate

CF:

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